# Optical eigenmodes in plane arrays of cylindrical waveguides. Analysis by means of multiple Mie scattering formalism and phenomenological model.

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## Abstract

We consider a plane periodical array of parallel cylindrical waveguides with evanescent coupling between them. A new method for calculating of isofrequency curves based on the multiple Mie scattering formalism (MMSF) is developed. This method is compared with the phenomenological model. The derivation of the phenomenological model by means of the MMSF is performed. The formulae for calculation of parameters of the phenomenological model are derived, such as propagation constants and coupling constant.

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## I. INTRODUCTION.

Nowadays, much attention is devoted to periodical arrays of evanescently coupled optical waveguides. Such systems represent the particular case of low-dimensional photonic crystal structures. The general feature of such systems is the existence of photonic band structure [1, 2] that is analogous to the electron band structure in solids. Therefore some effects in photonic crystal structures may be analogous to some phenomena in solids [3].

In this paper we consider a plane array of parallel equidistant waveguides. We assume that the interaction between the waveguides is enough weak but not negligible. In this case, the eigenmodes of the array may be represented in a spirit of tight binding method taken from the solid state physics. It means that the eigenmodes of the array can be expressed in terms of the eigenmodes of the noninteracting waveguides.

The eigenmodes for the $j$-th waveguide are described as follows [4]

$$\begin{aligned} \mathbf{E}_j(\mathbf{r}) &= e^{-i\omega t+i\beta_j z}\,\mathbf{U}_j(x_j,y_j), \\ \mathbf{H}_j(\mathbf{r}) &= e^{-i\omega t+i\beta_j z}\,\mathbf{V}_j(x_j,y_j), \end{aligned} \tag{1}$$

where $\omega$ is a frequency of an eigenmode, $x_j$, $y_j$, $z$ are the coordinates of a point $\mathbf{r}$ with respect to the axis of the waveguide, $\beta_j$ is the propagation constant of the $j$-th waveguide. If $\beta_j > \omega$ (the speed of light is assumed to be unit), the functions $\mathbf{U}_j(x_j,y_j)$, $\mathbf{V}_j(x_j,y_j)$ outside the waveguide decrease exponentially as the distance of the observation point from the waveguide increases. Thus, the mode is evanescent and it cannot be converted into a free photon.

So, the eigenmodes of the array of weakly interacting waveguides may be represented in following way:

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &\approx e^{-i\omega t}\sum_{j=1}^{N} A_j(z)\,\mathbf{U}_j(x_j,y_j), \\ \mathbf{H}(\mathbf{r}) &\approx e^{-i\omega t}\sum_{j=1}^{N} A_j(z)\,\mathbf{V}_j(x_j,y_j). \end{aligned} \tag{2}$$

If the distance between the waveguides is large enough, the coupling between only the nearest waveguides may be taken into account. Then, the equation for an eigenmode of the array reads

$$i\frac{dA_j}{dz}(z) + \beta_j A_j(z) + \gamma\Big(A_{j-1}(z) + A_{j+1}(z)\Big) = 0, \tag{3}$$

where $\gamma$ is a nearest neighbor coupling constant (for derivation see, for example, [4]). This equation or the analogous equations are usually used for simulation of optical effects in systems of interacting waveguides, such as optical Bloch oscillations [5–7], Zener tunneling [8–10], dynamic localization [11, 12], Anderson localization [13].

A principle drawback of Eq. (3) is that the phenomenological constants $\beta_j$ and $\gamma$ are unknown. They can be found from the experiment if one supposes that Eq. (3) is applicable to the system under study.

However, in the important particular case of the cylindrical waveguides, one may propose the exact description of the optical properties of the array. In this case every eigenmode is characterized by the angular momentum $m$. The eigenmodes are described as follows [4]

$$\begin{aligned} \mathbf{E}_{jm}(\mathbf{r}) &= e^{-i\omega t + im\phi_j + i\beta_{jm} z}\,\mathbf{U}_{jm}(\rho_j), \\ \mathbf{H}_{jm}(\mathbf{r}) &= e^{-i\omega t + im\phi_j + i\beta_{jm} z}\,\mathbf{V}_{jm}(\rho_j). \end{aligned} \tag{4}$$

Here $m$ is the angular momentum, $\rho_j$, $\phi_j$ are polar coordinates of a point $\mathbf{r}$ with respect to the axis of the waveguide, $U_{jm}(\rho_j),\ V_{jm}(\rho_j) \sim H_m(\varkappa_{jm}\rho_j)$, where $H_m(x)$ is a Hankel function of the first kind, $\varkappa_{jm} = \sqrt{\omega^2 - \beta_{jm}^2}$.

The rigorous formalism for description of array of cylindrical waveguides makes use of the exact solution for the electromagnetic wave scattering problem by an infinite cylinder. The proposed description is based on the possibility to generalize this solution for the case of many parallel cylinders — multiple Mie scattering formalism (MMSF) for the arrays of infinite cylinders [14–18]. This approach is similar to the multiple Mie scattering formalism for spherical particles [19–22]. The MMSF can be used for investigation of scattering and transmission of light by photonic crystals [16, 17] or irregular systems of cylindrical waveguides [15], for calculation of the eigenmode frequencies and band structures of a plane array of cylindrical waveguides [18] and of two-dimensional photonic crystals [17].

In this paper we use the MMSF for calculation of the isofrequency curves for the array of identical waveguides. We consider the case when the waveguides of the array are situated close to each other and ascertain if the phenomenological approach is applicable for such a system. For this purpose we derive the phenomenological approach from the MMSF and develop the method for calculation the coupling constant.

The MMSF is explained in Sect. II. In Sect. III we discuss the connection between the MMSF

and the phenomenological approach and explain how the coupling constant $\gamma$ in Eq. (3) can be calculated. In Sect. IV we calculate the isofrequency curves of a plane array of infinite cylindrical waveguides. We compare the isofrequency curves calculated by means of MMSF with that calculated by the phenomenological model. In Conclusion we discuss the possibility of further development of method used in this paper.

## II. MULTIPLE MIE SCATTERING FORMALISM.

Let us consider an array of $N$ parallel cylindrical dielectric rods. The axes of the rods are in the plane $y = 0$ and they all are parallel to the $z$-axis. The refractive index of the $j$-th array is denoted as $n_j$. Let the array is illuminated by the external field of the certain frequency $\omega$ and longitudinal wave vector $\beta$:

$$\mathbf{E}^{\text{ext}}(\mathbf{r}) = e^{-i\omega t+i\beta z}\ \mathbf{E}^{\text{ext}}(x,y), \qquad \mathbf{H}^{\text{ext}}(\mathbf{r}) = e^{-i\omega t+i\beta z}\ \mathbf{H}^{\text{ext}}(x,y). \tag{5}$$

This field causes the response of the array. The field inside the $j$-th rod is

$$\begin{aligned}\tilde{\mathbf{E}}_j(\mathbf{r}) &= e^{-i\omega t+i\beta z}\sum_m e^{im\phi_j}\Big(c_{jm}\,\mathbf{M}^1_{\omega_j\beta m}(\rho_j) - d_{jm}\,\mathbf{N}^1_{\omega_j\beta m}(\rho_j)\Big),\\ \tilde{\mathbf{H}}_j(\mathbf{r}) &= e^{-i\omega t+i\beta z}\sum_m e^{im\phi_j}\Big(c_{jm}\,\mathbf{N}^1_{\omega_j\beta m}(\rho_j) + d_{jm}\,\mathbf{M}^1_{\omega_j\beta m}(\rho_j)\Big), \qquad \rho_j < R.\end{aligned} \tag{6}$$

Here $\rho_j$, $\phi_j$, $z$ are the cylindrical coordinates of the point $\mathbf{r}$ respectively to the axis of the $j$-th waveguide, and $\omega_j = n_j\omega$. The functions $\mathbf{M}^1_{\omega_j\beta m}(\rho_j)$ and $\mathbf{N}^1_{\omega_j\beta m}(\rho_j)$ are linear superpositions of the Bessel functions. The partial amplitudes $c_{jm}$, $d_{jm}$ determine the field inside the $j$-th rod. Below the factor $e^{-i\omega t+i\beta z}$ is omitted, for short.

The field scattered by the $j$-th rod may be represented in the form

$$\begin{aligned}\mathbf{E}_j(\mathbf{r}) &= \sum_m e^{im\phi_j}\ \Big(a_{jm}\ \mathbf{M}^2_{\omega\beta m}(\rho_j) - b_{jm}\ \mathbf{N}^2_{\omega\beta m}(\rho_j)\Big),\\ \mathbf{H}_j(\mathbf{r}) &= \sum_m e^{im\phi_j}\ \Big(a_{jm}\ \mathbf{N}^2_{\omega\beta m}(\rho_j) + b_{jm}\ \mathbf{M}^2_{\omega\beta m}(\rho_j)\Big), \qquad \rho_j > R.\end{aligned} \tag{7}$$

The functions $\mathbf{M}^2_{\omega\beta m}(\rho_j)$ and $\mathbf{N}^2_{\omega\beta m}(\rho_j)$ are linear superpositions of the Hankel functions of the first kind for the imaginary argument.

On the other hand, the field Eq. (7) can be represented in the alternative form as an expansion in terms of functions $\mathbf{M}^1_{\omega\beta m}(\rho_l)$ and $\mathbf{N}^1_{\omega\beta m}(\rho_l)$ for any $l \neq j$:

$$
\begin{aligned}
\mathbf{E}_j(\mathbf{r}) &= \sum_m e^{im\phi_l}\ \Big(p^l_{jm}\ \mathbf{M}^1_{\omega\beta m}(\rho_l) - q^l_{jm}\ \mathbf{N}^1_{\omega\beta m}(\rho_l)\Big), \\
\mathbf{H}_j(\mathbf{r}) &= \sum_m e^{im\phi_l}\ \Big(p^l_{jm}\ \mathbf{N}^1_{\omega\beta m}(\rho_l) + q^l_{jm}\ \mathbf{M}^1_{\omega\beta m}(\rho_l)\Big), \qquad l \neq j.
\end{aligned} \tag{8}
$$

Let us emphasize that the Eqs. (7) and (8) represent the same field, i. e. the field scattered by the $j$-th waveguide.

According to [15], one can relate the amplitudes $p^l_{jm}$, $q^l_{jm}$ and $a_{lm}$, $b_{lm}$ as follows

$$
p^l_{jm} = \sum_{n=-\infty}^{+\infty} U^{ln}_{jm}(\omega,\beta)\ a_{ln}, \qquad q^l_{jm} = \sum_{n=-\infty}^{+\infty} U^{ln}_{jm}(\omega,\beta)\ b_{ln}, \tag{9}
$$

where

$$
U^{ln}_{jm}(\omega,\beta) = H_{n-m}(\varkappa a\cdot|j-l|)\times \begin{cases} 1 & \text{if } l>j, \\ (-1)^{m-n} & \text{if } l<j, \end{cases} \tag{10}
$$

$\varkappa = \sqrt{\omega^2-\beta^2}$ and $H_m(x)$ is the Hankel function of the first kind.

Let us introduce a notation

$$
\begin{aligned}
\mathbf{E}'_j(\mathbf{r}) &= \sum_{l\neq j}\mathbf{E}_l(\mathbf{r}) = \sum_{l\neq j}\sum_m e^{im\phi_j}\ \Big(p^l_{jm}\mathbf{M}^1_{\omega\beta m}(\rho_j) - q^l_{jm}\ \mathbf{N}^1_{\omega\beta m}(\rho_j)\Big), \\
\mathbf{H}'_j(\mathbf{r}) &= \sum_{l\neq j}\mathbf{H}_l(\mathbf{r}) = \sum_{l\neq j}\sum_m e^{im\phi_j}\ \Big(p^l_{jm}\ \mathbf{N}^1_{\omega\beta m}(\rho_j) + q^l_{jm}\mathbf{M}^1_{\omega\beta m}(\rho_j)\Big), \qquad \rho_j > R.
\end{aligned} \tag{11}
$$

One can rewrite it in the form

$$
\begin{aligned}
\mathbf{E}'_j(\mathbf{r}) &= \sum_{l\neq j}\mathbf{E}_l(\mathbf{r}) = \sum_m e^{im\phi_j}\ \Big(p_{jm}\,\mathbf{M}^1_{\omega\beta m}(\rho_j) - q_{jm}\,\mathbf{N}^1_{\omega\beta m}(\rho_j)\Big), \\
\mathbf{H}'_j(\mathbf{r}) &= \sum_{l\neq j}\mathbf{H}_l(\mathbf{r}) = \sum_m e^{im\phi_j}\ \Big(p_{jm}\,\mathbf{N}^1_{\omega\beta m}(\rho_j) + q_{jm}\,\mathbf{M}^1_{\omega\beta m}(\rho_j)\Big), \qquad \rho_j > R.
\end{aligned} \tag{12}
$$

Here

$$
\begin{aligned}
p_{jm} &= \sum_{l\neq j} p^l_{jm} = \sum_{l\neq j}\sum_{n=-\infty}^{+\infty} U^{ln}_{jm}(\omega,\beta)\,a_{ln}, \\
q_{jm} &= \sum_{l\neq j} q^l_{jm} = \sum_{l\neq j}\sum_{n=-\infty}^{+\infty} U^{ln}_{jm}(\omega,\beta)\,b_{ln}.
\end{aligned} \tag{13}
$$

Let us assume that the external field

$$
\begin{aligned}
\mathbf{E}^{\mathrm{ext}}(\mathbf{r}) &= \sum_m \; e^{im\phi_j}\Big(P_m^j\,\mathbf{M}^1_{\omega\beta m}(\rho_j) - Q_m^j\,\mathbf{N}^1_{\omega\beta m}(\rho_j)\Big),\\
\mathbf{H}^{\mathrm{ext}}(\mathbf{r}) &= \sum_m \; e^{im\phi_j}\Big(P_m^j\,\mathbf{N}^1_{\omega\beta m}(\rho_j) + Q_m^j\,\mathbf{M}^1_{\omega\beta m}(\rho_j)\Big).
\end{aligned}
\tag{14}
$$

Then, the field outside of the array may be represented in the form

$$
\begin{aligned}
\mathbf{E}(\mathbf{r}) &= \mathbf{E}^{\mathrm{ext}}(\mathbf{r}) + \mathbf{E}_j(\mathbf{r}) + \sum_{l\neq j}\mathbf{E}_l(\mathbf{r}),\\
\mathbf{H}(\mathbf{r}) &= \mathbf{H}^{\mathrm{ext}}(\mathbf{r}) + \mathbf{H}_j(\mathbf{r}) + \sum_{l\neq j}\mathbf{H}_l(\mathbf{r}),
\end{aligned}
\tag{15}
$$

where the number $j$ is arbitrary.

The relations between fields outside and inside the $j$-th rod follow from the boundary conditions on its surface. These relations take the following form:

$$
\begin{pmatrix} a_{mj} \\ b_{mj} \end{pmatrix} = \hat{S}_{jm}(\omega,\beta)\;\begin{pmatrix} P_m^j + p_{jm} \\ Q_m^j + q_{jm} \end{pmatrix}, \tag{16}
$$

$$
\begin{pmatrix} c_{mj} \\ d_{mj} \end{pmatrix} = \hat{T}_{mj}(\omega,\beta)\;\begin{pmatrix} a_{mj} \\ b_{mj} \end{pmatrix}. \tag{17}
$$

Taking into account Eq. (13) in Eq. (16) one obtains the self-consistent system of equations

$$
\hat{S}^{-1}_{jm}(\omega,\beta)\;\begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix} - \sum_{l\neq j}^{N}\;\sum_{n=-\infty}^{+\infty} U^{ln}_{jm}(\omega,\beta)\;\begin{pmatrix} a_{ln} \\ b_{ln} \end{pmatrix} = \begin{pmatrix} P_m^j \\ Q_m^j \end{pmatrix}. \tag{18}
$$

The system of equation Eq. (18) describes the response of the array on the external electromagnetic field, determined by the amplitudes $P_m^j$, $Q_m^j$. At the same time, if one takes $P_m^j = Q_m^j = 0$, the Eq. (18) describes the electromagnetic eigenmodes for the array under consideration. These modes are described as follows:

$$
\hat{S}^{-1}_{jm}(\omega,\beta)\;\begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix} - \sum_{l\neq j}^{N}\;\sum_{n=-\infty}^{+\infty} U^{ln}_{jm}(\omega,\beta)\;\begin{pmatrix} a_{ln} \\ b_{ln} \end{pmatrix} = 0. \tag{19}
$$

The homogeneous system (19) possesses a nontrivial solution only if

$$
\det\;\left|\hat{S}^{-1}_{jm}(\omega,\beta)\;\delta_{jl}\;\delta_{mn} - U^{ln}_{jm}(\omega,\beta)\right| = 0. \tag{20}
$$

This equation allows to obtain the eigenvalues of $\beta$ for the eigenmodes of the array.

In particular, for the single rod this equation takes the form

$$\det \left|\hat{S}^{-1}_{jm}(\omega,\beta)\right| = 0. \tag{21}$$

The solutions of this equation $\beta_{jm}$ (depending on $\omega$) are the propagation constants of the $j$-th waveguide, that is assumed noninteracting with the other waveguides. One can see that these propagation constants are characterized by the angular momentum $m$, as it was mentioned above.

Below we apply Eq. (18) and Eq. (19) to describe the optical properties of the array of the rods.

## III. RELATIONSHIP OF THE MULTIPLE SCATTERING FORMALISM AND THE PHENOMENOLOGICAL APPROACH.

Let us derive the simplified equations which describe the optical properties of the array of the rods under consideration.

Every rod is characterized by a set of its propagation constants $\beta_{jm}$, satisfying to Eq. (21). Let us notice that the propagation constants corresponding to the opposite angular momenta coincide, i. e. $\beta_{jm} = \beta_{j,-m}$.

We suppose that the propagation constants of different waveguides differ slightly. Besides, the coupling $U^{ln}_{jm}(\omega,\beta)$ is weak and may be considered as a perturbation with respect to $\hat{S}^{-1}_{jm}(\omega,\beta)$. Therefore, we can consider the optical excitations originated from the propagation constants with fixed angular momentum $m$. Two cases are possible: $m = 0$ and $m \neq 0$. For the first case one should take into account two partial amplitudes $a_{j0}$, $b_{j0}$. For the second case the system of equations should include four partial amplitudes $a_{jm}$, $b_{jm}$ and $a_{j,-m}$, $b_{j,-m}$, since the propagation constants for the angular momenta $m$ and $-m$ coincide.

Below we take into account only the coupling between the nearest neighbors, since the coupling is evanescent.

### A. First case: $m = 0$.

The first case is $m = 0$. In this case the main system of equations takes the form

$$\hat{S}_{j0}^{-1}(\omega,\beta)\begin{pmatrix}a_{j0}\\ b_{j0}\end{pmatrix}-\sum_{l=j\pm1}U_{j0}^{l0}(\omega,\beta)\begin{pmatrix}a_{l0}\\ b_{l0}\end{pmatrix}=0. \tag{22}$$

Below we omit the arguments $\omega$, $\beta$ for short.

The matrix $\hat{S}_{j0}^{-1}$ is diagonal,

$$\hat{S}_{j0}^{-1}=\begin{pmatrix}A_j & 0\\ 0 & B_j\end{pmatrix}. \tag{23}$$

Here $A_j$, $B_j$ are some functions of $\omega$ and $\beta$. So, Eq. (22) separates in two independent systems of equations:

$$A_j\ a_{j0}-\sum_{l=j\pm1}U_{j0}^{l0}\ a_{l0}=0, \tag{24}$$

$$B_j\ b_{j0}-\sum_{l=j\pm1}U_{j0}^{l0}\ b_{l0}=0, \tag{25}$$

Each propagation constant $\beta_{j0}$ satisfy to one of the following equations:

$$A_j(\omega,\beta_{j0})=0, \qquad B_j(\omega,\beta_{j0})=0. \tag{26}$$

Below we consider Eq. (24) only, since for Eq. (25) the derivation is similar.

Since the coupling between the waveguides is weak, the isofrequency curve originating from any propagation constant is narrow. Therefore one can represent $A_j$ in following way:

$$A_j=D_j^A\times(\beta-\beta_{j0}). \tag{27}$$

Here $\beta_{j0}$ satisfies to the first of Eqs. (26).

Substituting this to Eq. (24), we obtain:

$$(\beta-\beta_{j0})\ a_{j0}-\sum_{l=j\pm1}\frac{U_{j0}^{l0}}{D_j^A}\ a_{l0}=0. \tag{28}$$

Let us notice that $U_{j0}^{j-1,0}=U_{j0}^{j+1,0}$. Here we assume that $U_{j0}^{l0}(\omega,\beta)=U_{j0}^{l0}(\omega,\beta_{j0})$ and that the value $U_{j0}^{j\pm1,0}(\omega,\beta_{j0})/D_j^A$ is independent on $j$. So, introducing the notation

$$\gamma=\frac{U_{j0}^{j\pm1,0}(\omega,\beta_{j0})}{D_j^A}, \tag{29}$$

we obtain

$$\left(\beta - \beta_{j0}\right)\, a_{j0} - \gamma \left(a_{j-1,0} + a_{j+1,0}\right) = 0. \tag{30}$$

The Eq. (30) possesses the nontrivial solutions only for eigenvalues of $\beta$.

The electric field outside the array of waveguides is

$$\mathbf{E}(t, \mathbf{r}) = e^{-i\omega t} \sum_{j=1}^{N} \sum_{\beta} \; e^{i\beta z}\; a_{j0}(\beta)\; \mathbf{M}^2_{\omega\beta 0}(\rho_j). \tag{31}$$

The expression for the magnetic field is analogous.

Here $\sum\limits_{\beta}$ means the sum over the eigenvalues of $\beta$. We have added the argument $\beta$ to the partial amplitudes $a_{j0}$, $b_{j0}$, since the partial amplitudes depend on the eigenvalue $\beta$.

Since the rods differ slightly and the interaction between them is weak, all the eigenvalues of $\beta$ are close to each other. So, one can suppose that $\mathbf{M}^2_{\omega\beta 0}(\rho_j) = \mathbf{M}^2_{\omega\beta_{j0} 0}(\rho_j)$. Let us introduce the notation

$$a_{j0}(z) = \sum_{\beta} \; e^{i\beta z}\; a_{j0}(\beta). \tag{32}$$

So, the equation (31) takes the form:

$$\mathbf{E}(t, \mathbf{r}) = e^{-i\omega t} \sum_{j=1}^{N} \; a_{j0}(z)\; \mathbf{M}^2_{\omega\beta_{j0} 0}(\rho_j). \tag{33}$$

Taking into account Eq. (30), one can write the equation for $a_{j0}(z)$:

$$\left(i\frac{d}{dz} + \beta_{j0}\right)\; a_{j0}(z) + \gamma \left(a_{j-1,0}(z) + a_{j+1,0}(z)\right) = 0. \tag{34}$$

This equation coincides to Eq. (3).

In the similar way one can derive the equation

$$\left(i\frac{d}{dz} + \beta_{j0}\right)\; b_{j0}(z) + \gamma \left(b_{j-1,0}(z) + b_{j+1,0}(z)\right) = 0. \tag{35}$$

where

$$\gamma = \frac{U^{j\pm 1,0}_{j0}(\omega, \beta_{j0})}{D^B_j}, \tag{36}$$

and $D^B_j$ is determined by the equation

$$B_j = D_j^B \times (\beta - \beta_{j0}). \tag{37}$$

## B. Second case: $m \neq 0$.

The second case is $m \neq 0$. As it was mentioned above, we should take into account the partial amplitudes $a_{jm}$, $b_{jm}$ and $a_{j,-m}$, $b_{j,-m}$ both. So, the main system of equations takes the form

$$\begin{aligned}
&\hat{S}_{jm}^{-1} \begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix} - \sum_{l=j\pm1} \left\{ U_{jm}^{lm} \begin{pmatrix} a_{lm} \\ b_{lm} \end{pmatrix} + U_{jm}^{l,-m} \begin{pmatrix} a_{l,-m} \\ b_{l,-m} \end{pmatrix} \right\} = 0, \\
&\hat{S}_{j,-m}^{-1} \begin{pmatrix} a_{j,-m} \\ b_{j,-m} \end{pmatrix} - \sum_{l=j\pm1} \left\{ U_{j,-m}^{l,-m} \begin{pmatrix} a_{l,-m} \\ b_{l,-m} \end{pmatrix} + U_{j,-m}^{lm} \begin{pmatrix} a_{lm} \\ b_{lm} \end{pmatrix} \right\} = 0.
\end{aligned} \tag{38}$$

It is convenient to introduce the following notations:

$$\begin{aligned}
&U_{jm}^{j-1,m} = U_{jm}^{j+1,m} = U_{j,-m}^{j-1,-m} = U_{j,-m}^{j+1,-m} = U_m, \\
&U_{jm}^{j-1,-m} = U_{jm}^{j+1,-m} = U_{j,-m}^{j-1,m} = U_{j,-m}^{j+1,m} = V_m.
\end{aligned} \tag{39}$$

Substituting this to Eq. (38), one gets

$$\begin{aligned}
&\hat{S}_{jm}^{-1} \begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix} - \sum_{l=j\pm1} \left\{ U_m \begin{pmatrix} a_{lm} \\ b_{lm} \end{pmatrix} + V_m \begin{pmatrix} a_{l,-m} \\ b_{l,-m} \end{pmatrix} \right\} = 0, \\
&\hat{S}_{j,-m}^{-1} \begin{pmatrix} a_{j,-m} \\ b_{j,-m} \end{pmatrix} - \sum_{l=j\pm1} \left\{ U_m \begin{pmatrix} a_{l,-m} \\ b_{l,-m} \end{pmatrix} + V_m \begin{pmatrix} a_{lm} \\ b_{lm} \end{pmatrix} \right\} = 0.
\end{aligned} \tag{40}$$

Below we show that there are two types of solutions.

Let us suppose that the partial amplitudes $a_{jm}$, $b_{jm}$ and $a_{j,-m}$, $b_{j,-m}$ are connected by the following relation:

$$\begin{pmatrix} a_{j,-m} \\ b_{j,-m} \end{pmatrix} = \hat{M} \begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix}. \tag{41}$$

Substituting this to (40), one gets:

$$
\begin{gathered}
\hat{S}_{jm}^{-1}\begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix} - \sum_{l=j\pm1}\left(U_m + V_m\,\hat{M}\right)\begin{pmatrix} a_{lm} \\ b_{lm} \end{pmatrix} = 0, \\
\hat{M}^{-1}\,\hat{S}_{j,-m}^{-1}\hat{M}\begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix} - \sum_{l=j\pm1}\left(U_m + V_m\,\hat{M}^{-1}\right)\begin{pmatrix} a_{lm} \\ b_{lm} \end{pmatrix} = 0.
\end{gathered}
\tag{42}
$$

Both equations in (42) should coincide. Therefore the matrix $\hat{M}$ should satisfy to following conditions:

$$
\begin{gathered}
\hat{M}^{-1} = \hat{M}, \\
\hat{M}^{-1}\,\hat{S}_{j,-m}^{-1}\,\hat{M} = \hat{S}_{jm}^{-1}.
\end{gathered}
\tag{43}
$$

To find the possible forms of matrix $\hat{M}$ one should use a relation between the matrices $\hat{S}_{jm}$ and $\hat{S}_{j,-m}$. These matrices possess the form

$$
\hat{S}_{jm}^{-1} = \begin{pmatrix} iA & C \\ -C & iB \end{pmatrix}, \qquad \hat{S}_{j,-m}^{-1} = \begin{pmatrix} iA & -C \\ C & iB \end{pmatrix},
\tag{44}
$$

where $A$, $B$ and $C$ are some real functions of $\omega$ and $\beta$.

So, one can find easily, that there are only two possible forms of the matrix $\hat{M}$:

$$
\hat{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \text{or} \qquad \hat{M} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.
\tag{45}
$$

So, we see that the solutions of equation (38) separates in two different types.

For the solutions of the first type

$$
a_{j,-m} = a_{jm}, \qquad b_{j,-m} = -b_{jm},
\tag{46}
$$

and $a_{jm}$, $b_{jm}$ satisfy the equation

$$
\hat{S}_{jm}^{-1}\begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix} - \sum_{l=j\pm1}\begin{pmatrix} U_m + V_m & 0 \\ 0 & U_m - V_m \end{pmatrix}\begin{pmatrix} a_{lm} \\ b_{lm} \end{pmatrix} = 0.
\tag{47}
$$

For the solutions of the second type

$$
a_{j,-m} = -a_{jm}, \qquad b_{j,-m} = b_{jm},
\tag{48}
$$

and $a_{jm}$, $b_{jm}$ satisfy the equation

$$\hat{S}_{jm}^{-1} \begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix} - \sum_{l=j\pm 1} \begin{pmatrix} U_m - V_m & 0 \\ 0 & U_m + V_m \end{pmatrix} \begin{pmatrix} a_{lm} \\ b_{lm} \end{pmatrix} = 0. \tag{49}$$

Below we consider an equation

$$\hat{S}_{jm}^{-1} \begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix} - \sum_{l=j\pm 1} \hat{W}_m \begin{pmatrix} a_{lm} \\ b_{lm} \end{pmatrix} = 0, \tag{50}$$

where

$$\begin{aligned} \hat{W}_m &= \begin{pmatrix} U_m + V_m & 0 \\ 0 & U_m - V_m \end{pmatrix} \qquad \text{for the first case,} \\ \hat{W}_m &= \begin{pmatrix} U_m - V_m & 0 \\ 0 & U_m + V_m \end{pmatrix} \qquad \text{for the second case.} \end{aligned} \tag{51}$$

Let $\mathbf{u}_{jm} = \begin{pmatrix} \tilde{a}_{jm} \\ \tilde{b}_{jm} \end{pmatrix}$ be the solution of the equation

$$\hat{S}_{jm}^{-1}(\omega, \beta_{jm})\ \mathbf{u}_{jm} = 0. \tag{52}$$

Remain that $\beta_{jm}$ satisfies to the equation $\det \hat{S}_{jm}^{-1}(\omega, \beta_{jm}) = 0$. The vector $\mathbf{u}_{jm}$ is one of the two eigenvectors for matrix $\hat{S}_{jm}^{-1}(\omega, \beta_{jm})$ possessing a vanishing zero eigenvalue. Let $\mathbf{v}_{jm}$ be the other eigenfunction for the matrix $\hat{S}_{jm}^{-1}(\omega, \beta_{jm})$, with $\mu_{jm}$ being the corresponding eigenvalue. Thus,

$$\hat{S}_{jm}^{-1}(\omega, \beta_{jm})\ \mathbf{v}_{jm} = \mu_{jm}\, \mathbf{v}_{jm}. \tag{53}$$

The vectors $\mathbf{u}_{jm}$, $\mathbf{v}_{jm}$ are linearly independent. One assumes that $\mathbf{u}_{jm}^{\dagger}\, \mathbf{u}_{jm} = \mathbf{v}_{jm}^{\dagger}\, \mathbf{v}_{jm} = 1$. Let us find a solution of Eq.(47) in the form

$$\begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix} = A_{jm}\, \mathbf{u}_{jm} + B_{jm}\, \mathbf{v}_{jm}. \tag{54}$$

Since the coupling of the adjacent waveguides is a small perturbation to $\hat{S}_{jm}^{-1}$ in Eq. (47), the vector $\begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix}$ is practically "parallel" to $\mathbf{u}_{jm}$. For this reason, $|B_{jm}| \ll |A_{jm}|$. Within the perturbation approach, the value $\beta - \beta_{jm}$ is a small parameter. Then,

$$\hat{S}_{jm}^{-1}(\omega, \beta) \approx \hat{S}_{jm}^{-1}(\omega, \beta_{jm}) + (\beta - \beta_{jm})\, \hat{D}_{jm}, \tag{55}$$

where $\hat{D}_{jm}$ is the derivative of the matrix $\hat{S}_{jm}^{-1}(\omega, \beta)$ taken at the point $\beta = \beta_{jm}$. Substituting (54) and (55) to (50), one gets:

$$\begin{aligned} &\left\{ \hat{S}_{jm}^{-1}(\omega, \beta_{jm}) + (\beta - \beta_{jm})\, \hat{D}_{jm} \right\} \Big( A_{jm}\, \mathbf{u}_{jm} + B_{jm}\, \mathbf{v}_{jm} \Big) - \\ &\quad - \sum_{l=j\pm 1} \hat{W}_m \Big( A_{lm}\, \mathbf{u}_{lm} + B_{lm}\, \mathbf{v}_{lm} \Big) = 0. \end{aligned} \tag{56}$$

With Eq. (52) being taken into account, the first order perturbation approach gives

$$\mu_{jm}\, B_{jm}\, \mathbf{v}_{jm} + (\beta - \beta_{jm})\, A_{jm}\, \hat{D}_{jm}\, \mathbf{u}_{jm} - \sum_{l=j\pm 1} A_{lm}\, \hat{W}_m\, \mathbf{u}_{jm} = 0. \tag{57}$$

It is convenient to introduce a vector $\mathbf{w}_{jm}$ completely defined by the conditions:

$$\begin{aligned} \mathbf{w}_{jm}^{\dagger}\, \hat{D}_{jm}\, \mathbf{u}_{jm} &= 1, \\ \mathbf{w}_{jm}^{\dagger}\, \mathbf{v}_{jm} &= 0. \end{aligned} \tag{58}$$

Multiplying Eq. (57) by $\mathbf{w}_{jm}^{\dagger}$ results in the equation

$$(\beta - \beta_{jm})\, A_{jm} - \sum_{l=j\pm 1} A_{lm}\ \mathbf{w}_{jm}^{\dagger}\, \hat{W}_m(\beta)\, \mathbf{u}_{jm} = 0, \tag{59}$$

If the variation of $\beta_{jm}$ is small as $j$ changes, the variation of the product $\mathbf{w}_{jm}^{\dagger}\, \hat{W}_m(\beta)\, \mathbf{u}_{jm}$ is small, as well. Therefore, one can neglect its dependence on $j$. Denoting

$$\gamma = \mathbf{w}_{jm}^{\dagger}\, \hat{W}_m(\beta)\, \mathbf{u}_{jm}. \tag{60}$$

one obtains:

$$(\beta - \beta_{jm})\ A_{jm} - \gamma\, \Big( A_{j-1,m} + A_{j+1,m} \Big) = 0. \tag{61}$$

The electric field outside the array is

$$\mathbf{E}(t,\mathbf{r}) = e^{-i\omega t} \sum_{j=1}^{N} \sum_{\beta} e^{i\beta z} \left\{ e^{im\phi_j} \left( a_{jm}(\beta) \mathbf{M}^2_{\omega\beta m}(\rho_j) - b_{jm}(\beta) \mathbf{N}^2_{\omega\beta m}(\rho_j) \right) + \right.$$
$$\left. + e^{-im\phi_j} \left( a_{j,-m}(\beta) \mathbf{M}^2_{\omega\beta,-m}(\rho_j) - b_{j,-m}(\beta) \mathbf{N}^2_{\omega\beta,-m}(\rho_j) \right) \right\}. \tag{62}$$

The expression for the magnetic field is analogous. Here the sum over $\beta$ means the sum over the eigenvalues of longitudinal wave vector of the array. We have added the argument $\beta$ to partial amplitudes since they may be different for different eigenmodes of the array.

In the first approximation,

$$\begin{pmatrix} a_{jm}(\beta) \\ b_{jm}(\beta) \end{pmatrix} = \begin{pmatrix} \pm a_{j,-m}(\beta) \\ \mp b_{j,-m}(\beta) \end{pmatrix} = A_{jm}(\beta)\ \mathbf{u}_{jm}, \tag{63}$$

where the upper sign is for the first case and the lower sign for the second case.

The vector $\mathbf{u}_{jm}$ doesn't depend on $\beta$. The eigenvalues $\beta$ differ slightly, so one can suppose that $\mathbf{M}^2_{\omega\beta,\pm m}(\rho_j) \sim \mathbf{M}^2_{\omega\beta_{jm},\pm m}(\rho_j)$, $\mathbf{N}^2_{\omega\beta,\pm m}(\rho_j) \sim \mathbf{N}^2_{\omega\beta_{jm},\pm m}(\rho_j)$. Introducing the notation

$$A_{jm}(z) = \sum_{\beta} e^{i\beta z}\ A_{jm}(\beta), \tag{64}$$

one gets

$$\mathbf{E}(t,\mathbf{r}) = e^{-i\omega t} \sum_{j=1}^{N} A_{jm}(z) \left\{ \tilde{a}_{jm} \left( e^{im\phi_j}\ \mathbf{M}^2_{\omega\beta_{jm}m}(\rho_j) \pm e^{-im\phi_j}\ \mathbf{M}^2_{\omega\beta_{jm}-m}(\rho_j) \right) + \right.$$
$$\left. + \tilde{b}_{jm} \left( e^{im\phi_j}\ \mathbf{N}^2_{\omega\beta_{jm}m}(\rho_j) \mp e^{-im\phi_j}\ \mathbf{N}^2_{\omega\beta_{jm}-m}(\rho_j) \right) \right\}. \tag{65}$$

From Eqs. (61) and (64) it follows

$$\left( i\frac{d}{dz} + \beta_{jm} \right) A_{jm}(z) + \gamma \left( A_{j-1,m}(z) + A_{j+1,m}(z) \right) = 0. \tag{66}$$

This equation coincides to Eq. (3).

## IV. APPLICATION FOR ISOFREQUENCY CURVES CALCULATION.

Consider an infinite array of identical waveguides. The optical eigenmodes in this system possess the form of Bloch waves:

$$\mathbf{E}(t,\mathbf{r}) = e^{-i\omega t + i\beta z + ikx}\ \mathbf{U}(\mathbf{r}),$$
$$\mathbf{H}(t,\mathbf{r}) = e^{-i\omega t + i\beta z + ikx}\ \mathbf{V}(\mathbf{r}), \tag{67}$$

where $\mathbf{U}(\mathbf{r})$, $\mathbf{V}(\mathbf{r})$ are the periodical functions relatively to the coordinate $x$. Here $k$ is the transverse quasi wave vector belonging to the interval $-\pi < k \le \pi$ (here the period of the array is assumed to be unit). For the fixed frequency $\omega$ the longitudinal wave vector $\beta$ is connected with the transverse quasi wave vector $k$, and the function $\beta(k)$ is the so-called isofrequency curve.

For the field outside the waveguides Eq. (67) results in the relations for the partial amplitudes

$$a_{jm} = a_m\ e^{ikja}, \qquad b_{jm} = b_m\ e^{ikja}. \tag{68}$$

For the case of periodical array of identical waveguides the scattering matrices for all the waveguides are the same. Besides, the coupling coefficients $U_{jm}^{ln}(\omega,\beta)$ depend on $j-l$. So the system (18) takes the form

$$\hat{S}_m^{-1}(\omega,\beta)\ \begin{pmatrix} a_{jm} \\ b_{jm} \end{pmatrix} + \sum_{l=-\infty}^{+\infty} \sum_n\ U_m^n\Big(\omega,\beta,\ (l-j)a\Big)\ \begin{pmatrix} a_{ln} \\ b_{ln} \end{pmatrix} = 0. \tag{69}$$

Substituting (68) to (69), one obtains

$$\hat{S}_m^{-1}(\omega,\beta)\ \begin{pmatrix} a_m \\ b_m \end{pmatrix} + \sum_n\ U_m^n(\omega,\beta,k)\ \begin{pmatrix} a_n \\ b_n \end{pmatrix} = 0, \tag{70}$$

where

$$U_m^n(\omega,\beta,k) = \sum_{l=-\infty}^{+\infty}\ U_m^n\Big(\omega,\beta,\ (l-j)a\Big)\ e^{ik(l-j)a}. \tag{71}$$

In the matrix form this system of equations can be written as

$$\hat{L}(\omega,\beta,k)\ \mathbf{x} = 0, \tag{72}$$

where the matrix $\hat{L}(\beta,k)$ contains the scattering matrices $\hat{S}_m^{-1}(\omega,\beta)$ and the coupling coefficients $U_m^n(\omega,\beta,k)$, and the column vector $\mathbf{x}$ contains the partial amplitudes $a_m$, $b_m$.

The nontrivial solutions of system (70) exists when

$$\det\ \hat{L}(\omega,\beta,k) = 0. \tag{73}$$

For fixed frequency $\omega$ this equation implicitly determines the isofrequency curves $\beta(k)$.

The isofrequency curves can be derived also from the phenomenological model. Consider the equations (30) and (61). Assuming $A_{jm} = e^{ikaj}\, A_m$ and substituting this to (61), one immediately gets an explicit expression for isofrequency curves:

$$\beta(k) = \beta_m + 2\gamma\ \cos ka. \tag{74}$$

Here $\beta_m$ is the propagation constant corresponding to the angular momenta $m$ and $-m$ (it is independent on the number $j$ of a waveguide since all the waveguides are identical). The similar result can be obtained from Eq. (30) after substitution $a_{j0} = e^{ikaj}\, a_0$.

As it was mentioned above, for $m \neq 0$ the system (38) possesses two types of solutions with two different coupling constants $\gamma$. So, the propagation constant $\beta_m$, $m \neq 0$ gives rise to two different isofrequency curves. For the case $m = 0$ the propagation constant gives rise to one isofrequency curve only.

Here we compare the isofrequency curves calculated by means of the phenomenological model with the results of the rigorous model based on the multiple scattering formalism. I. e. we compare the results of calculations based on equations (73) and (74).

We take the array of waveguides of refractive index $n_{\rm wg} = 1.554$, and the refractive index of the medium outside the waveguides is $n_{\rm med} = 1.457$. We suppose that the period of the array is unit, $a = 1$. The waveguides are supposed to be situated close to each other, i. e. the radii of the waveguides are $R = 0.5$. The velocity of light in vacuum is assumed $c = 1$.

Below we use the multiple Mie scattering formalism for calculating several isofrequency curves originating from different propagation constants. The obtained isofrequency curves are compared with the prediction of the phenomenological model. To calculate the coupling constants $\gamma$ we use the formulae obtained in Sec. III.

For angular momentum $m = 0$ we take two propagation constants: $\beta_1 = 126.671$ and $\beta_2 = 126.704$. The coupling constants for them are $\gamma_1 = -3.92 \times 10^{-2}$ and $\gamma_2 = -3.78 \times 10^{-2}$.

The obtained isofrequency curves are presented in Fig. 1 and Fig. 2. The results of calculation by means of MMSF are presented by dots, and the predictions of the phenomenological model are shown by the solid curves. The horizontal lines show the propagation constants. (Here and below the isofrequency curves are plotted for $0 < k < \pi$ since the function $\beta(k)$ is even, $\beta(-k) = \beta(k)$.)

One can see that the isofrequency curves obtained by MMSF and the phenomenological model almost coincide.

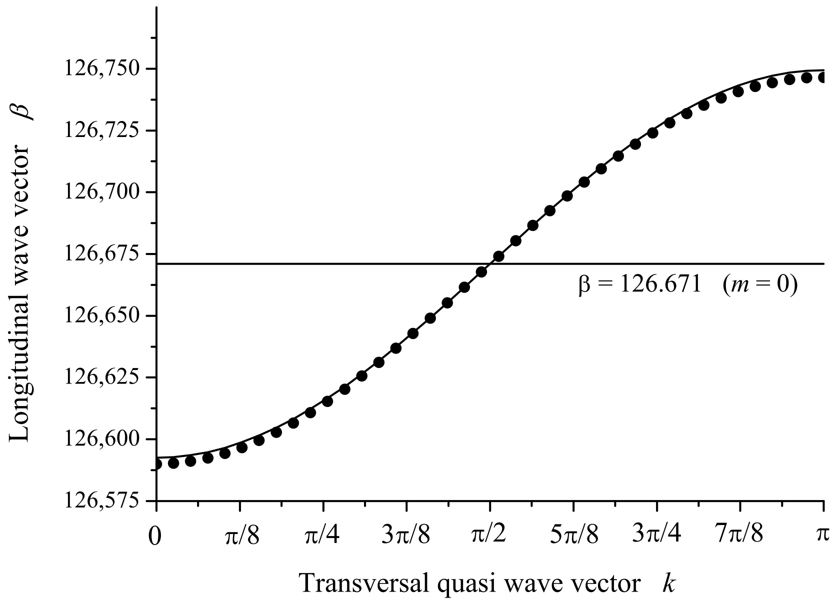


Figure 1: Isofrequency curve originating from the propagation constant $\beta = 126.671$ $(m = 0)$. Dots for the curve obtained by MMSF, solid line for the curve obtained by phenomenological model.

For the angular momentum $m = 1$ we take two propagation constants also: $\beta_3 = 131.099$ and $\beta_4 = 132.0092$. For every of propagation constants two coupling constants exist. For $\beta_3$ the coupling constants are $\gamma_3' = 9.51 \times 10^{-3}$ and $\gamma_3'' = 5.97 \times 10^{-3}$. For $\beta_4$ they are $\gamma_4' = 7.90 \times 10^{-4}$ and $\gamma_4'' = 7.12 \times 10^{-4}$.

The obtained isofrequency curves are represented in Fig. 3 and Fig. 4. One can see that the agreement between the results of MMSF and phenomenological model for the angular momentum $m = 1$ is much worth then for $m = 0$. In spite of this, the phenomenological model is applicable for the qualitative description of the isofrequency curves.

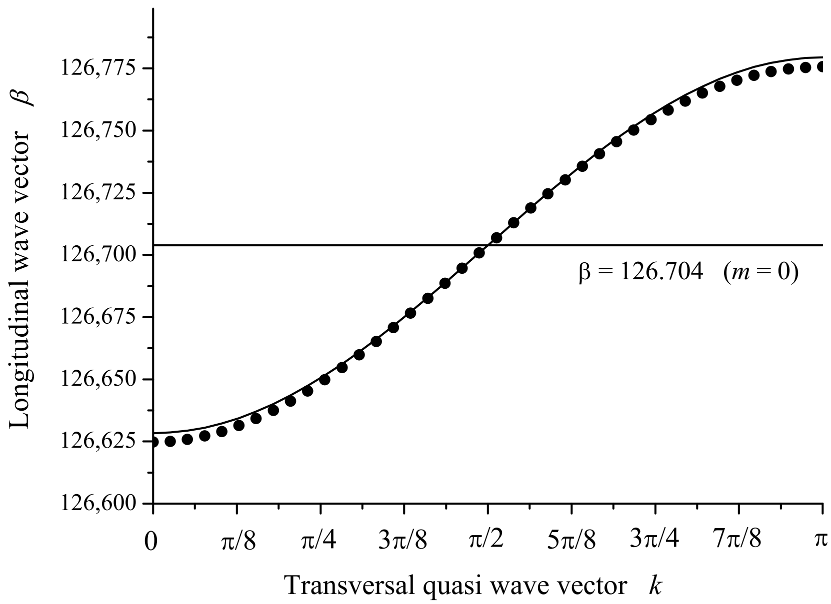


Figure 2: Isofrequency curve originating from the propagation constant $\beta = 126.704$ ($m = 0$). Dots for the curve obtained by MMSF, solid line for the curve obtained by phenomenological model.

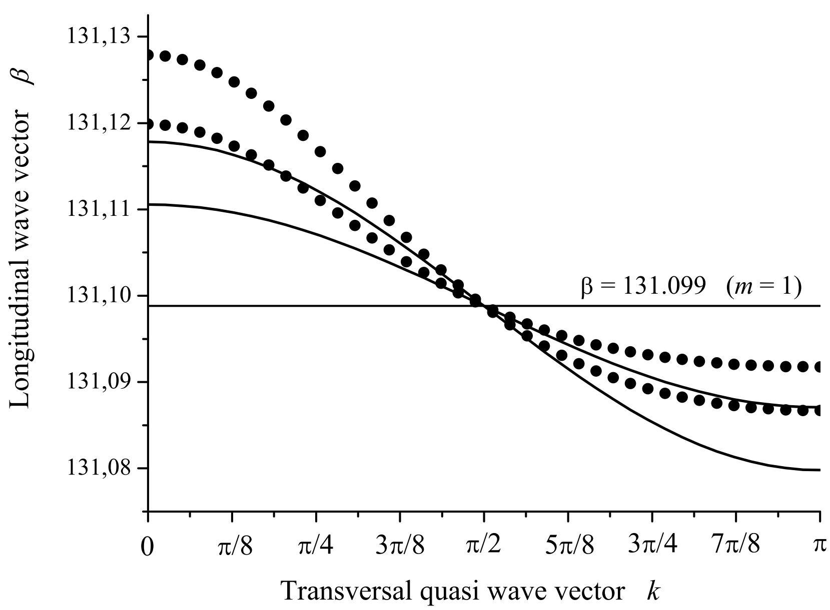


Figure 3: Isofrequency curves originating from the propagation constant $\beta = 131.099$ ($m = 1$). Dots for the curves obtained by MMSF, solid lines for the curves obtained by phenomenological model.

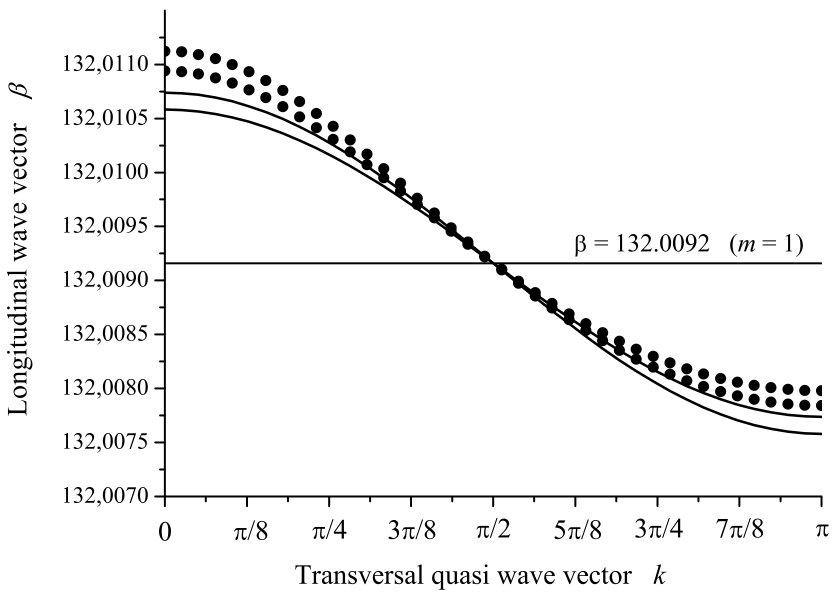


Figure 4: Isofrequency curves originating from the propagation constant $\beta = 132.0092$ ($m = 1$). Dots for the curves obtained by MMSF, solid lines for the curves obtained by phenomenological model.

## V. CONCLUSION.

In this paper we considered the planar arrays of cylindrical rods by means of two methods: the phenomenological model and the multiple Mie scattering formalism.

The MMSF has several advantages over the phenomenological method based on Eq. (3). First, the MMSF allows to calculate the behavior of the optical excitation for the case of the strong coupling between the waveguides, while the phenomenological method is applicable only for the case of the weak coupling. Second, the input data for the MMSF are the geometrical properties of the array and refractive indices of waveguides, while the phenomenological method requires some data that should be obtained experimentally, such as the propagation constants of waveguides and coupling constants.

We demonstrated that for the case of evanescent coupling of rods the phenomenological model can be derived from MMSF. We developed the method to calculate the propagation constants $\beta_{jm}$ and coupling constants $\gamma$. The applicability of the developed method is demonstrated for different isofrequency curves. The method represented in this work allows to produce the numerical simulation without need of experimental investigation of components of optical devices.

The method developed in this paper was used for isofrequency curves calculation for the case of weak interaction between the waveguides only. But it may be useful also for the systems with strong coupling between the waveguides. In this case the hybridization of modes with different angular momenta may take place due to the coupling. Mathematically it means that one can't neglect the coupling coefficients $U_{jm}^{ln}(\omega, \beta)$ with $n \neq m$. In this situation the isofrequency curves may possess the shape much more complicated than the phenomenological model predicts.

The MMSF represented in this paper is convenient only for the waveguides of cylindrical form, because in this case the scattering matrix can be calculated easily. However, this method can be applied for the waveguides of another shape, but in this case it would be more difficult to calculate the scattering matrix. Besides, the scattering by noncylindrical waveguides would mix the harmonics with different angular momenta. Mathematically it means that the scattering matrix $\hat{S}(\omega, \beta)$ contains some "nondiagonal" elements describing the transition of harmonics $e^{im\phi_j}\,\mathbf{M}^1_{\omega\beta m}(\rho_j)$, $e^{im\phi_j}\,\mathbf{N}^1_{\omega\beta m}(\rho_j)$ to harmonics $e^{in\phi_j}\,\mathbf{M}^2_{\omega\beta n}(\rho_j)$, $e^{in\phi_j}\,\mathbf{N}^2_{\omega\beta n}(\rho_j)$ with $n \neq m$. Due to the existence of nonzero "nondiagonal" elements, the scattering matrix $\hat{S}(\omega, \beta)$ can't be separated to several matrices $\hat{S}_m(\omega, \beta)$ with fixed $m$.

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